Round 1: Probability and Statistics

1. OBJECTIVES

- To develop fluency in probability and statistics.
- To develop mathematical reasoning.

2. DESCRIPTION & RATIONALE

Probability and statistics are necessarily taught for exam purposes as a series of processes. These techniques were developed and are used by professionals for find models to capture the variability in (normally very large) data sets and tools to model variables which have a random element. The two parts of this round consist of a dice game in which students make probabilistic judgements and an activity in which they need to identify models for data sets.

1. The game of Hedgehog. The game involves rolling two dice. If you roll 1, 2, 3, 4, 5 on both dice you score the total of the two dice. If you roll a 6 on either of the two dice, your turn ends but you lose all the points you scored in that turn. If you roll a double 6, your turn ends and you lose all the points you scored in total up to that point resetting your score to zero. Or you can choose to pass, ending the turn without rolling. The first to score 50 points wins. Teams line up and take a turn to play a game against one player from the opposing team. There must be no communication (verbal or non-verbal) between the active players and those waiting. A judge will keep score. When one team has scored 50 the final scores are recorded. Points will be awarded according to the scores with a bonus for winning (roughly 20% of the points for wining and 80% for the points scored).

2. COU Shut-the-Box. The game is set up with twelve counters for each team, each set numbered 1 to 12, set out in order on a playing grid, with two columns for each team. Teams take it in turns to roll two dice. They can combine the two numbers with any arithmetic operation to make a whole number score. They announce this with the calculation e.g. if they roll a 3 and a 4, they can score 4-3=1, 4+3=7, 4x3=12, but not 4÷3 as this is not a whole number. They choose and announce one of these. Now they move counters with numbers that sum to the chosen score forward to the second column. These numbers are now 'shut'. For example, from the start, if the player chose the score of 12, they could move forward (i) the 12 counter or (ii) the 10 counter and the 2 counter or (iii) the 3 counter, the 4 counter and the 5 counter or many other possibilities. The player must complete their turn from taking the dice to moving the counters within 30 seconds. If not, they cannot move and play passes to the other team.

   They can move as many counters as they like as long as they total to the score they announced. Play now moves to player 1 in the opposing team, who play with their own set. Next is player 2 of the first team, who again rolls the two dice, announces the calculation for their score and moves counters totalling this score. However, they can only move counters which are not already shut. Play continues with teams taking turns and each player in each team taking a turn. The aim is to 'shut-the-box' which means all of the

counters have been shut. However, if in any turn, a player rolls dice for which no possible score allows counters to be moved, then their game is over and the first such team have lost. The other team can then continue until either they shut-the-box or they cannot go.

At this point scoring is as follows: score the total of the shut counters, plus a 5-point bonus for winning plus a 10-point bonus if you shut-the-box. See the sample game in the student guide for more details.

3. Data – Chart – Analysis. At the end of the Hedgehog round, teams will be handed an envelope containing a set of cards and an answer sheet. $\frac{1}{3}$ will contain details of data sets, $\frac{1}{3}$ will show statistical charts and $\frac{1}{3}$ will contain a summary analysis. Just before the start of the algebra round they will need to hand in their completed answer sheet for marking, so they can work on this at any time. The total number of cards will depend on the round and the content but have been either 18 or 24 cards in the past. When team members are not involved in the Gridlines and Target Maths rounds they could be doing this. They will need to organise the cards into sets with a best match of data + chart + analysis and write the card letters in 6 sets onto the answer sheet. The student workbook contains a sample set of cards to give an idea of the sorts of sets that could be used.

4. Mathematical Puzzling. In the Data-Chart-Analysis pack there will be a collection of additional sheets together with a corresponding answer sheet. These will contain a set of mathematical puzzles to solve. It will not be possible to prepare for this component as the puzzles will be chosen for their variety of content and type. Notably, there will not be any requirement to know any specific maths beyond arithmetic. Any member of the team can attempt any of these puzzles at any time when they are not otherwise engaged playing Gridlines Geometry or the Target Maths.

3. SCHOOL ACTIVITIES

In Class

The game of Hedgehog can be easily played in class. The important things is to consider strategy. It is one version of the classic 'Game of Pig'. There are online simulators for the game of pig, which let you see the probabilities at given points of the game to consider your strategy. For example: http://cs.gettysburg.edu/projects/pig/piggame.html
After students become expert in deploying strategies in this game, they can modify the variables in the game to see how this affects their strategy. For example, change the target score to a higher or lower amount. Or change the rules for example, make it so a 1 on either dice ends your turn but you keep the turn score. Of course, they cannot control random events and so the strategy has to be framed in terms of probabilities and the language of better/worse, more/less likely.

The game of shut-the-box is a classic game. Conventionally it is played in a wooden box with numbered wooden tabs which are moved to indicate being shut. There will either be tabs numbered 1 to 9 or as we are using, 1 to 12. It is played in different formats and with different rules. The Wikipedia article gives and excellent overview: en.wikipedia.org/wiki/Shut_the_box. The standard game is simpler than the COU version, notably when the two dice are rolled only the sum of the dice can be used as the score. We recommend playing the standard game only using the 1 to 9 counters to get started. Then extend to counters 1 to 12. Then allow add and subtract for the two dice to create a score. Then introduce multiplications and division. The most significant shift is including multiplication. Play one against one until players are very familiar with the game. Develop a coherent strategy. Finally introduce team play, where the further difficulty is to keep the strategy consistent across the whole team. There are lots of online simulators and indeed phone apps to practice, although none use the full COU rules. It provides an excellent site for discussing sophisticated probabilities. What is the probability in the standard game of shutting the whole box? Students should justify their strategies in terms of the probabilities of different future moves.

Data-Chart-Analysis will be drawn from reports about real world events or situations. In the students guide the charts are taken from a Channel 4 article summarising analysis on climate change data. The full report is here: www.channel4.com/news/factcheck/climate-change-in-ten-graphs. The issue is how the data can be used to make the inference that is given in the analysis and how the chart is illustrating this. This activity is complementary to the more direct method of taking small data sets, calculating statistics and drawing charts, common in exams and text books. But we hope that it gives a helpful broader view. Taking interesting data sets from any source and asking what sorts of statements you could make using this data and how you could you illustrate this effectively for different purposes for example on social media, TV news, in academic papers or government or organisational reports. This link takes you directly to download an EU book of illustrated statistics that you can use to explore: https://bit.ly/3EhgV8R.

Clubs and Tournament

COU shut-the-box will be played on large floor mats in the tournament, with large, numbered counters on the floor. Acceptable versions can easily be made in school for in-school tournaments. As with the algebra round, the Chart-Data-Analysis and mathematical puzzling round cannot be practiced directly. Instead, for Chart-Data-Analysis we recommend that students find other reports and articles online on issues of interest where data has been used to draw conclusions and illustrated with charts and to look at the relationships between the three components. For mathematical puzzling there are endless books and web sites with traditional mathematical puzzles. We hope your students will find these interesting to explore.

4. MATERIALS

In the school equipment pack:

1. A bag of 20 ordinary dice for two dice Hedgehog.
2. Two two-player 1-12 shut-the-box sets.

In the student guide:

3. A sample set of Data-Chart-Analysis cards for practice.
4. A page of mathematical puzzles to give a flavour of the sort of puzzles that will be included. However, they should not be seen as indicative of any specific puzzles they will receive.

5. AT THE COUNT ON US SECONDARY CHALLENGE

Two-dice Hedgehog and COU shut-the-box will take place simultaneously. Six teams will be called out to play school v school at three tables each with its own judge. They will play a round of games in which each team member plays one game against a different team member from an opposing school. Play needs to be near continuous, with minimal pausing between rolls. If the judge determines any undue delay or indeed any verbal or non-verbal communication between players, they will first issue a warning and on a repeat occasion will award the game to the other team. The other six teams will go to two COU shut-the-box 'courts' where they will play a mini tournament where each school plays the others. So, they will play two games, starting first in one of them.

The Data-Chart-Analysis envelope will be handed out as soon as all teams have completed the Hedgehog round. Answer sheets must be ready for collection later in the tournament, at the same time that the start of the algebra round is announced.

The Mathematical Puzzling activities will be inside the Data-Chart-Analysis envelope. The completed answer sheet will be handed in together with that for Data-Chart-Analysis.